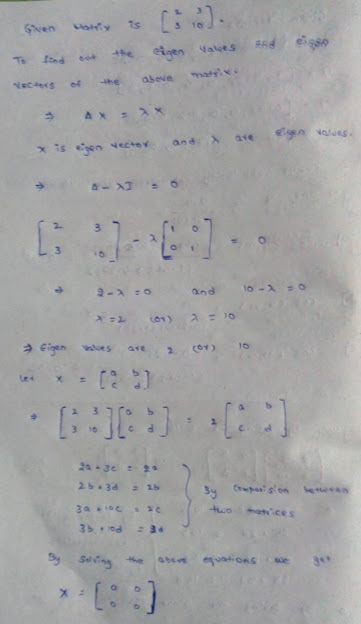
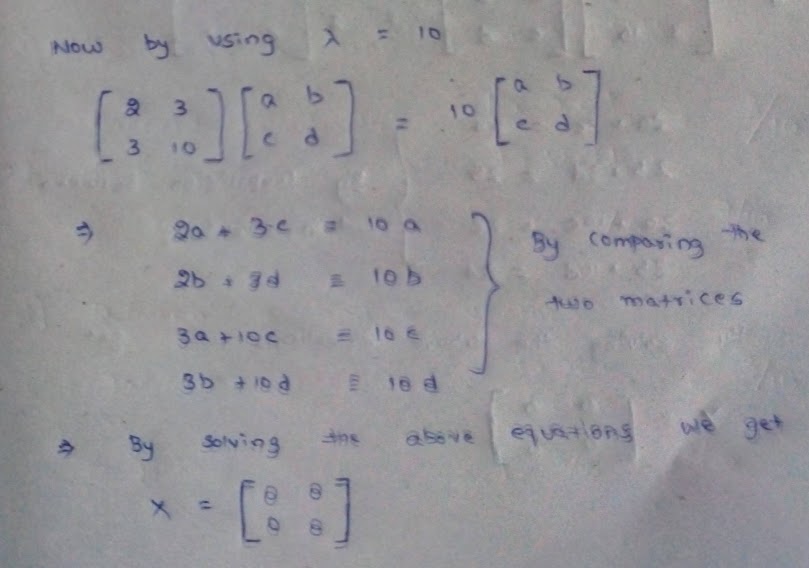
### **Dimensionality Reduction**

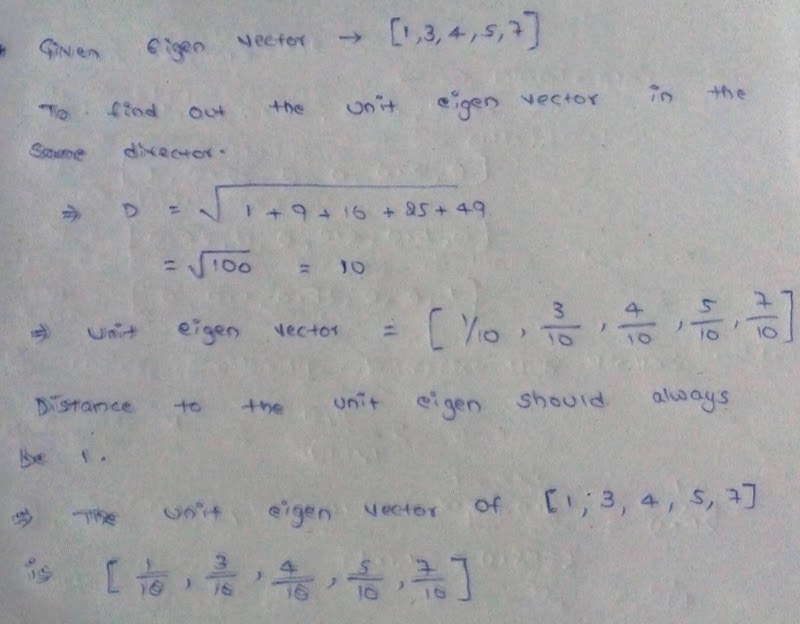
**Question 1**: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7, 3/7, 6/7], and another is [6/7, 2/7, -3/7]. Let the third column be [x, y, z]. Since the length of the vector [x, y, z] must be 1 there is a constraint that x2 + y2 + z2 = 1. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

**Question 2**: Find the eigenvalues and eigenvectors of the following matrix: You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

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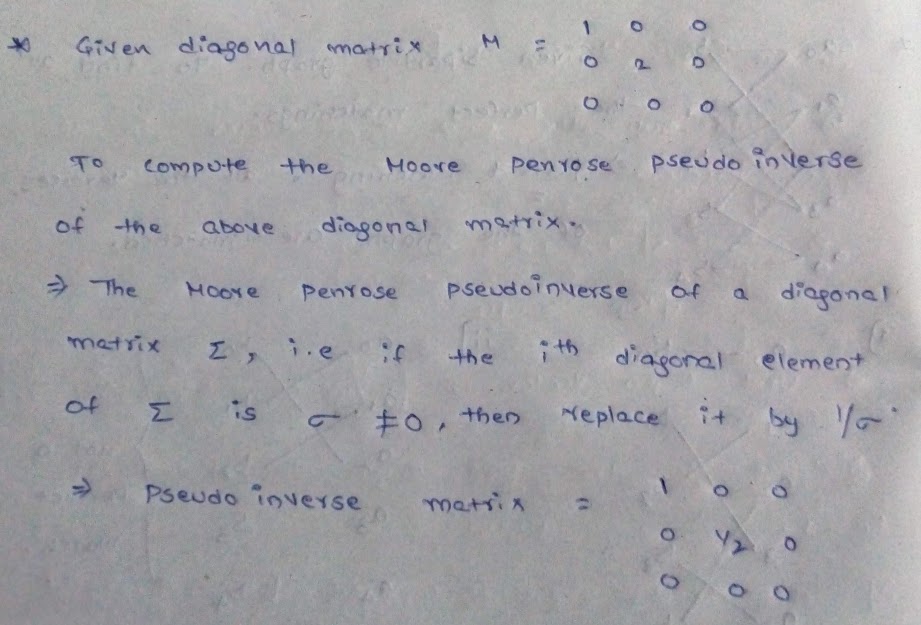
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**Question 3**: Suppose [1, 3, 4, 5, 7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.



**Question 4**: Suppose we have three points in a two dimensional space: (1, 1), (2, 2), and (3, 4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify its elements.

**Question 5**: Consider the diagonal matrix M = Compute its Moore-Penrose pseudoinverse.

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**Question 6**: When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose: Calculate the probability distribution for the rows.